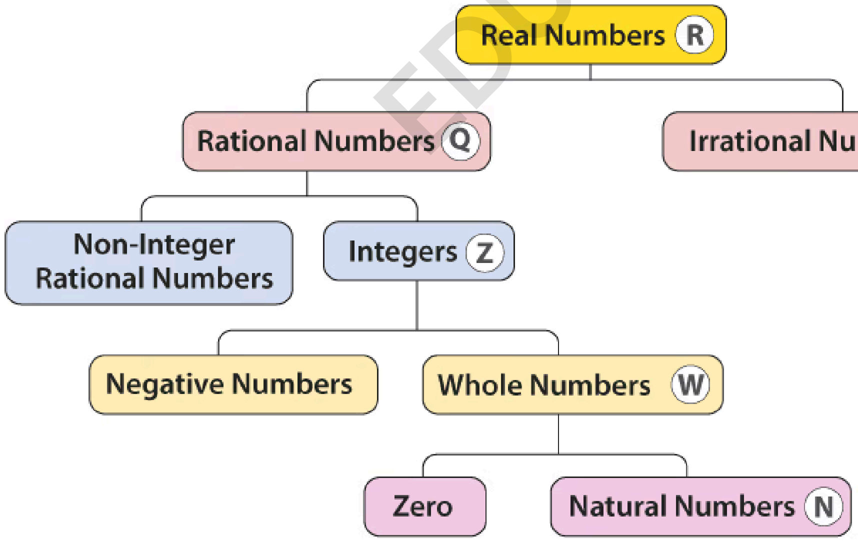
Chapter Notes: Real Numbers

Real Numbers

Real numbers are all the numbers that can be found on the number line. This includes both **rational numbers** (like 7, -3, 0.5, and 4/3) and **irrational numbers** (like √2) . They encompass integers, fractions, and decimals, representing a continuous, unbroken set of values.



Rational numbers such as integers (-2, 0, 1), fractions (1/2, 2.5) and irrational numbers such as √3, π(22/7), etc., are all real numbers.

Classification of Real Numbers

1. **Natural Numbers:** Natural Numbers are a set of counting numbers. They are denoted by N.
   N = {1, 2, 3, 4.........∞}
2. **Whole Numbers:** Whole numbers are a set of natural numbers plus zero.
   W = {0, 1, 2, 3 .... ∞}
3. **Integers:** Integers is a set of whole numbers and negative of all natural numbers.
   Z = { -3, -2, -1, 0, 1, 2, 3}
4. **Rational Numbers:** All the numbers that can be written in the p/q form where p and q are integers and q ≠ 0 are called rational numbers.
   E.g. 8/11, -3/17
5. **Irrational Numbers:** All the numbers that cannot be written in the p/q form are called irrational numbers. All the non-terminating and non-repeating decimal numbers are irrational numbers.
   E.g. √5, √3, √5 + √3, π

Fundamental Theorem of Arithmetic

To understand the fundamental theorem of Arithmetic, first, it is important to know what are composite numbers and prime numbers.

Composite Number

Composite Numbers are those numbers that have at least one factor other than one and the number itself.

Consider a number, 10. Now, the factors of 10 are 1, 2, 5 and 10. So it is a composite number.

Prime Number

Prime Numbers are those numbers that have exactly two factors: 1 and the number itself.

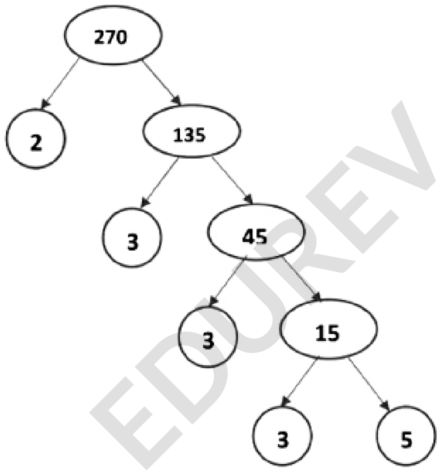
Let us take one more number 23. Now, the factors of 23 are 1 and 23. That means it has two factors 1 and the number itself, which is called a prime number.

Prime and Composite Numbers

> **Theorem:** Fundamental Theorem of Arithmetic states that every composite number can be expressed as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

Any **composite number** can be written as a product of primes in one way only as long as we are not particular about the order in which the primes occur.

Let us consider an example here: we will use a tree diagram to show the factors of 270.



$270 = 2 \times 3^3 \times 5$

Here, in the prime factorization of 270, the prime numbers appearing in both cases are the same, only the order in which they appear is different.

Therefore, the prime factorization of 270 is unique except for the order in which the primes occur.

Example 1: Check whether 15n can end with the digit zero for any natural number n.

Sol:

If a number ends with the digit 0, then it is divisible by both 2 and 5.

But prime factors of 15 are 3 and 5.

$15^n = (3 \times 5)^n = 3^n \times 5^n$

Here, the prime factorization of 15^n contains only 5 but not 2.

The uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the prime factorization of 15^n.

Therefore, 15^n cannot end with the digit zero for any natural number n.

Example 2: Explain, why (7 × 6 × 5 × 4 × 3 × 2 × 1) + 5 and (3 × 5 × 13 × 46) + 23 is a composite number?

Sol:

Composite numbers are those numbers that have at least one factor other than 1 and the number itself. E.g., 4, 6, 9

(i) $(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) + 5 = 5040 + 5 = 5045$

$5045 = 5 \times 1009$

As the factors of 5045 are 5 and 1009, it is a composite number.

(ii) $(3 \times 5 \times 13 \times 46) + 23 = 8970 + 23 = 8993$

$8993 = 17 \times 23 \times 23 \times 2$

As the factors of 8993 are 17, 23, and 2, it is a composite number.

HCF and LCM by Prime Factorisation Method

In this method, we first express the given numbers as a product of prime factors separately. Then, HCF is the product of the smaller power of each common prime factor in the numbers, and LCM is the product of the greatest power of each prime factor involved in the numbers.

For any two positive integers a and b,

HCF (a, b) × LCM (a, b) = a × b

Example 3: Find the LCM and HCF of 120 and 144 by the fundamental arithmetic theorem.

Sol:

$120 = 2^3 \times 3 \times 5$

$144 = 2^4 \times 3^2$

Now, HCF is the product of the smallest power of each common prime factor in the numbers.

Common Prime Factors	Smallest Power of Prime Factor
2	2^3
3	3^1

HCF (120, 144) = $2^3 \times 3 = 8 \times 3 = 24$

Prime Factors of 120 and 144	Greatest Power of Prime Factor
2	2^4
3	3^2
5	5^1

LCM is the product of the greatest power of each prime factor involved in the numbers.

LCM (120,144) = $2^4 \times 3^2 \times 5 = 16 \times 9 \times 5 = 720$

Example 4: If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$, where a, b are prime numbers, find the LCM (p, q).

Sol:

Given: $p = ab^2$ and $q = a^3b$

Prime Factors of p and q	Greatest Power of Prime Factor
a	a^3
b	b^2

LCM is the product of the greatest power of each prime factor involved in the numbers.

LCM (p, q) = $a^3 \times b^2 = a^3b^2$

Example 5: Write the HCF and LCM of the smallest odd composite number and the smallest odd prime number.

Sol:

The smallest odd composite number is 9, and the smallest odd prime number is 3.

$9 = 3^2$

$3 = 3^1$

Now, the smallest power of the common prime factor is 3^1.

HCF (9, 3) = 3

The greatest power of the common prime factor is 3^2.

LCM (9, 3) = $3^2 = 9$

Example 6: If HCF (253,440) = 11 and LCM (253,440)= 253 × R. Find the value of R.

Sol:

We know that,

HCF (a, b) × LCM (a, b) = a × b

∴ HCF (253, 440) × LCM (253, 440) = 253 × 440

11 × 253 × R = 253 × 440

R = 253 x 440 / 253 x 11

R = 40

Example 7: Ravi and Shikha drive around a circular sports field. Ravi takes 16 min to complete one round, while Shikha completes the round in 20 min. If both start at the same point, at the same time, and go in the same direction, then how much time will they meet at the starting point?

Sol:

Time taken by Ravi to drive one round of the circular field = 16 min. Time taken by Shikha to drive one round of the circular field =20 min.

The time after which they will again meet at the starting point will be equal to the LCM of 16 min and 20 min.

$16 = 2^4$

$20 = 2^2 \times 5$

LCM(16, 20) = $2^4 \times 5 = 16 \times 5 = 80$

Therefore, Ravi and Shikha will meet again at the starting point after 80 min.

Try yourself: According to the Fundamental Theorem of Arithmetic, which of the following statements is true?

a. Every prime number can be expressed as a product of composites.

b. Every composite number can be expressed as a product of primes, and this factorization is unique.

c. Every natural number can be expressed as a product of two prime numbers.

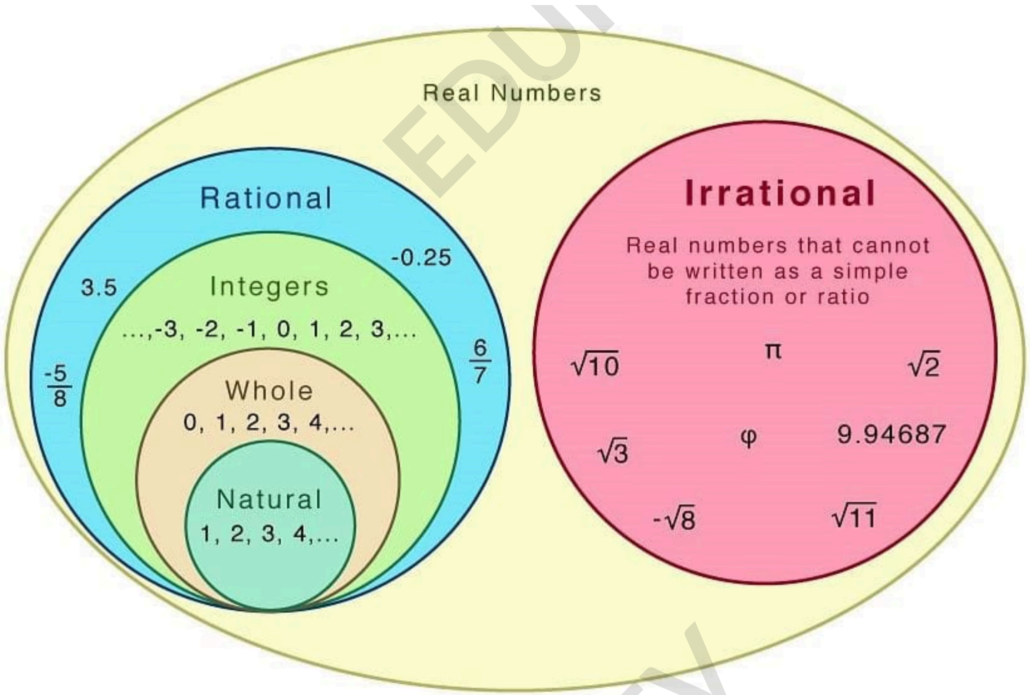
d. Every prime number can be expressed as a product of other prime numbers.

View Solution

Revisiting Irrational Numbers

Irrational numbers are those numbers that cannot be written in the form p/q, where p and q are integers and q ≠0. E.g.,√2, √3, √15

The square roots of all the numbers do not give an irrational number.



For example, √2 is an irrational number, but √4 = 2, which is rational.

Therefore, the square roots of all prime numbers are irrational.

If p is a prime number, then √p is an irrational number.

Theorem : If a prime number p divides a^2, then p divides a, where a is a positive integer.

Proof: Every positive integer can be expressed as the product of primes. Let $a = p_1p_2p_3 \ldots p_n$ where $p_1p_2p_3 \ldots p_n$ are all the prime factors of a.

$$a^2 = (p_1p_2p_3 \ldots p_n)^2 = (p_1p_2p_3 \ldots p_n)(p_1p_2p_3 \ldots p_n) = (p_1^2p_2^2p_3^2 \ldots p_n^2)$$

It is given that p divides a^2. According to the Fundamental Theorem of Arithmetic, we can say that p is one of the prime factors of a^2.

According to the **Fundamental Theorem of Arithmetic**, the prime factorization of a natural number is unique. Now, the only prime factors of a^2 are $p_1p_2p_3 \ldots p_n$. Therefore, p is one of $p_1p_2p_3 \ldots p_n$. So, p is also a factor of a.

If p divides a^2, then p also divides a.

Let us consider a positive integer 12. Now, the factors of 12 are 2, 2, and 3. On squaring 12, we get:

$$12^2 = 144$$

The factors of 144 are 2, 2, 2, 2, 3, and 3. If one of the factors, let's say 3, divides 144, then this factor, 3, will also divide 144, as it is one of the factors of 144 also.

Theorem: Prove that √2 is an irrational number.

Sol:

Let us assume that $\sqrt{2}$ is rational. Since it is a rational number it can be expressed in the form a/b, where a and b are integers and $b \neq 0$.

Now, a and b have no common factor other than 1.

$\sqrt{2} = a/b$, where a and b are coprime.

On squaring both sides, we get:

$2 = a^2/b^2$

$2b^2 = a^2$

Therefore, 2 divides a^2.

We know that when 2 divides a^2, then 2 divides a also.

We can write $a = 2m$, where m is an integer.

Putting $a = 2m$ in $2b^2 = a^2$:

$2b^2 = (2m)^2$

$2b^2 = 4m^2$

$b^2 = 2m^2$

Again, if 2 divides b^2, then 2 divides b also.

Therefore, 2 is a common factor of a and b.

But this contradicts the fact that a and b have no common factor other than 1.

So, we conclude that $\sqrt{2}$ is irrational.

Example 8: Show that 3√2 is an irrational number.

Sol:

Let us assume that $3\sqrt{2}$ is rational.

Then $3\sqrt{2}$ can be expressed in the form a/b where a and b are integers and $b \neq 0$. Now, a and b have no common factor other than 1.

$3\sqrt{2} = a/b$, where a and b are coprime integers.

$\sqrt{2} = a/3b$

Since 3, a, and b are integers, $a/3b$ is rational.

Now, a rational number cannot be equal to an irrational number, that is $\sqrt{2}$.

So, we conclude that $3\sqrt{2}$ is irrational.

Example 9: Show that 5 – √3 is irrational.

Sol:

Let us assume that $5 - \sqrt{3}$ is rational. As it is a rational number, it can be expressed in the form a/b where a and b are integers and $b \neq 0$. Now, a and b have no common factor other than 1.

$5 - \sqrt{3} = a/b$, where a and b are coprime.

$5 - a/b = \sqrt{3}$

$5b - a = \sqrt{3}b$

As 5, a, and b are integers, $5b - a$ is rational.

Now, $\sqrt{3}$ is an irrational number and cannot be equal to a rational number.

So, we conclude that $5 - \sqrt{3}$ is irrational.